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What is Oiliness?[1]

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An analysis is made of the concept of oiliness and of the problems connected with the determination of the oiliness of oils. The major difficulty involves a separation of the effects of viscosity and oiliness since both effects enter into measurements made in practical devices and under practical conditions. On the basis of information obtained at the Bureau using several devices, a method is proposed for separating these two effects. This makes it possible therefore to investigate the effects of the pertinent variables on oiliness without the introduction of viscosity effects.

IN recent years, the term "oiliness" has gradually come into use to represent that characteristic of liquids which results in lowering of friction between surfaces moving relative to one another and which cannot be accounted for on the basis of viscosity. Unfortunately, the conditions under which this characteristic manifests itself are so complex that it has not been possible as yet to assign values to oiliness which are independent of the particular apparatus and procedure used. This raises the question as to whether oiliness is really a property of the lubricant, but the available information is still insufficient to provide a certain answer. Many theories have been advanced to account for oiliness, but none of them appear to be sufficiently comprehensive to cover all of the known facts, even though these facts are few in number. In view of this lack of basic information, it seems preferable to confine the present discussion to the experimental phase of the oiliness problem.

Since oiliness manifests itself through a lowering of friction between surfaces moving relative to one another, the simplest approach to an understanding of the problem is to discuss the experimental variables involved in the measurement of friction. Confining attention to a journal bearing, if the friction is measured at constant load over a wide speed range with a mineral and a fatty oil of the same viscosity and under such

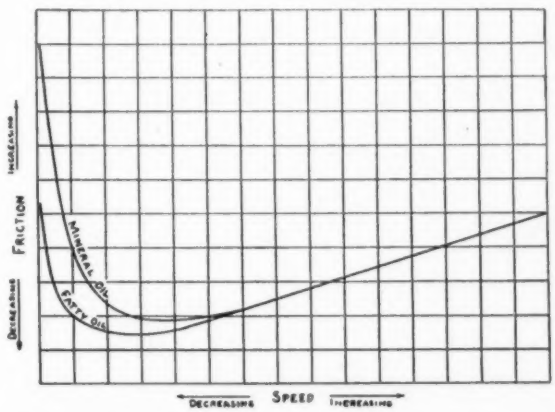

FIG. 1. Coefficient of friction-speed curves for mineral and fatty oils.

[1] Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce.

[2] Bureau of Standards, Washington, D. C.

conditions that the temperature in the oil film is identical in the two cases and constant at the various speeds, then the experimental data will be reproduced by curves of the type shown in Fig. 1. It is seen that, over part of the speed range, a lower friction is obtained with the fatty oil than with the mineral oil, but that the difference between the two curves varies with the speed and becomes negligible at the higher speeds.

According to Herschel,[3] who made a very thorough study of the problem several years ago, oiliness is "the property that causes a difference in the friction when two lubricants of the same viscosity at the temperature of the oil film are used under identical conditions." The curves in Fig. 1 satisfy the requirements of this definition, but it is observed that the oiliness effect varies with the speed.

An analysis of the definition immediately gives rise to the following questions:

(1) How does the value obtained for the oiliness of an oil change with the operating conditions of the device being used for measuring the friction?

(2) How is it possible to evaluate the change in oiliness of an oil with change in temperature, if the two oils have different temperature coefficients of viscosity?

(3) How is it possible to compare the oiliness of two oils of different viscosities?

Under the above definition, it is possible to obtain a partial answer to the first question, but any possibility of answering the last two questions is automatically eliminated. The main deficiency in the definition is that it limits the comparison to oils of the same viscosity, and accordingly it can only define a special case of the general concept of oiliness. The formulation of the general concept is difficult for it involves comprehensive information on the effects of variation in all of the variables concerned in the development of friction between surfaces moving relative to one another. In a thorough investigation of all of these variables lies the only rational basis for the formulation of a general definition of oiliness and for a method of determining it. In particular, some means of separating the effects of viscosity and oiliness is essential.

[3] Herschel, Journal of the Society of Automotive Engineers **10**, 31 (1922)

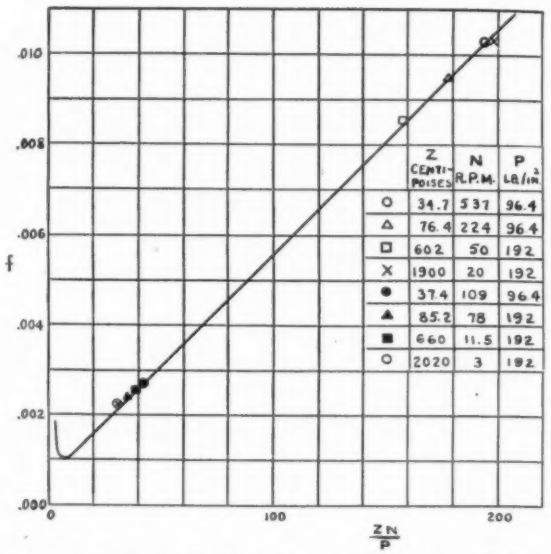

FIG. 2. Curve indicating the correlation of coefficient of friction data on a given journal bearing by use of the composite variable ZN/P.

Referring again to Fig. 1, it is seen that, at the high speeds, the friction first decreases as the speed decreases, but that it reaches a minimum value and then increases with further decrease in speed. It is customary to refer to the region on the right of the minimum point as that of thick film lubrication, while that to the left of the minimum point is called the region of thin film lubrication. In the thick film region, considerable information has been obtained which indicates that viscosity is the only property of the oil which affects the friction. Further, it has been possible to evaluate quantitatively the effects of speed and load. Thus the extensive experimental work of McKee[4] has shown conclusively that for any given journal bearing

$$f=a+k(ZN/P), \qquad (1)$$

where f is the coefficient of friction, namely the friction per unit load; Z is the oil viscosity in absolute units; N is the journal speed; P is the applied load per unit projected area; while a and k are constants, the latter depending upon the dimensions of the bearing. Experimental verification of Eq. (1) is given in Fig. 2. For any given load, it is seen that the coefficient of

[4] McKee, Trans. A. S. M. E., APM—51—15, p. 161, 1929; Mech. Eng. **51**, 593 (1929).

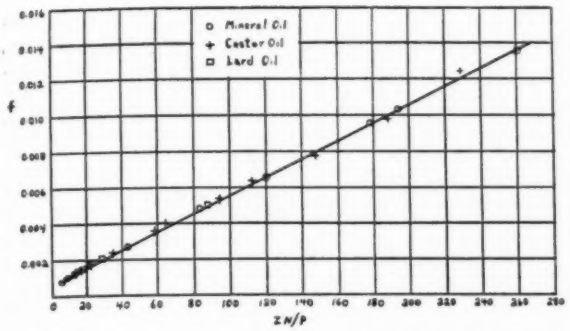

FIG. 3. Coefficient of friction curve illustrating the negligible effect of oiliness in the region of thick film lubrication.

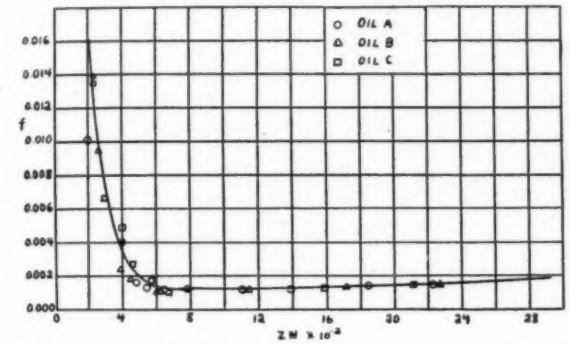

FIG. 4. Coefficient of friction curve illustrating a correlation of the data in the region of thin film lubrication by use of the composite variable ZN at constant load.

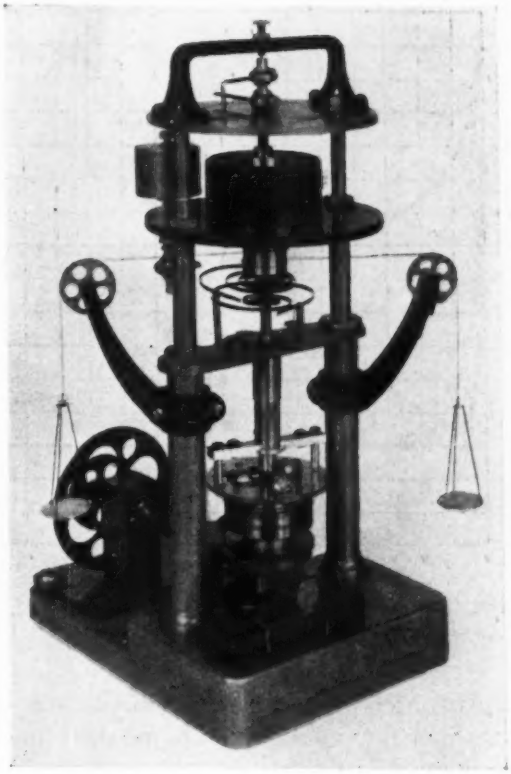

FIG. 5. Machine for measurement of coefficient of friction in the region of thin film lubrication.

friction varies directly with the viscosity and the speed. Thus if coefficient of friction-speed curves are obtained at the same load for a number of oils of different viscosities, a plot of coefficient of friction against the product of the speed and the viscosity will result in a single curve for all of the data. In the thick film region, therefore, use of the composite variable ZN at constant load, or of ZN/P with varying loads, permits a correlation of friction data for oils of different viscosities. That oiliness has no effect in this region is illustrated in Fig. 3, where data for lard, castor, and mineral oils are shown. There is no apparent effect due to the greater oiliness of lard and castor oils.

In order to determine whether use of the composite variable ZN would similarly correlate viscosity effects in the thin film region, friction measurements were made on several machines. Data were obtained first on several oils with a journal bearing machine[5] operating in the region just to the left of the minimum point on the coefficient of friction-speed curves. Results at one load are shown in Fig. 4, where it is seen that the variable ZN accounts for viscosity effects.[6] Data at other loads led to similar conclusions. This work was not conclusive since it was not possible to operate this machine under conditions which extended far into the region of thin film lubrication. Accordingly, further work was done on machines designed primarily for operation in this region.

A photograph of the machine used for most of the work is shown in Fig. 5, a detailed description of which may be found in a previous publication.[7] The essential part of the apparatus was the sliding of three steel rolls on the top of a flat ring raised above the surface of a rotating flat plate. For purposes of reference, this will be called the

[5] S. A. E. J. 31, T371 (1932).

[6] The Saybolt viscosities of the three oils at 130°F are as follows: Oil A—75 sec.; Oil B—139 sec.; and Oil C—230 sec.

[7] Proceedings, 13th Annual Meeting, A. P. I., Sect. 3, December, 1932; page 154.

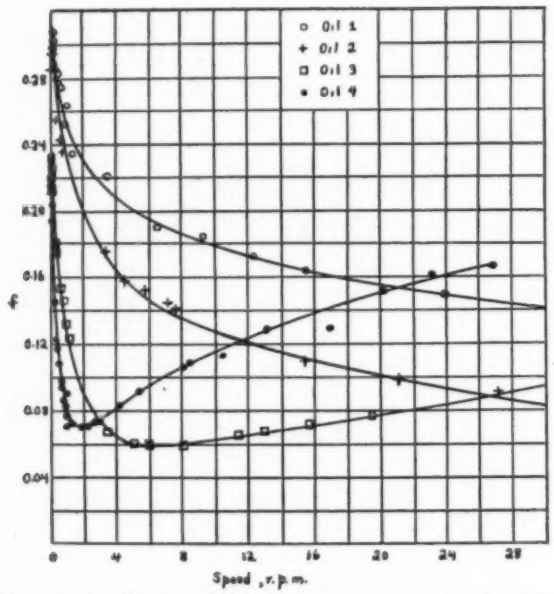

FIG. 6. Coefficient of friction-speed curves for 4 mineral oils. Data obtained with machine in Fig. 5.

flat plate friction machine. Extensive data were obtained on four mineral oils having the following viscosities at 25°C:

Oil	1	2	3	4
Viscosity (centipoises)	41.7	233	3120	9340

Coefficient of friction-speed curves for these oils are shown in Fig. 6, the same load being used in each case. At very low speeds, the coefficient of friction decreases as the viscosity increases, in contrast with the trend in the thick film region where the coefficient of friction increases as the viscosity increases. At 2 r.p.m. for example, the coefficient of friction for the oil of lowest viscosity (oil 1) is 0.230, while for oil 4, having a viscosity more than 200 times as great as oil 1, the coefficient of friction is 0.070. As the speed increases, the curves pass through minimum points and then intersect, so that no simple relation exists between coefficient of friction and viscosity in this range. In the case of oils 1 and 2, minimum points were found at higher speeds than those shown.

Following the procedure adopted with the journal bearing data, the coefficient of friction values shown in Fig. 6 were plotted against ZN, the product of the speed and the viscosity. The result is illustrated in Fig. 7 where it is observed that a single curve can be drawn through the points within experimental error. With this machine, similar data were obtained at other loads and with raised ring specimens of various materials. In every case, the same conclusion was reached, namely that for results with this machine use of the composite variable ZN accounts for the effect of viscosity.

No measurements have as yet been made with this machine at various temperatures in order to ascertain whether the ZN plot will correlate

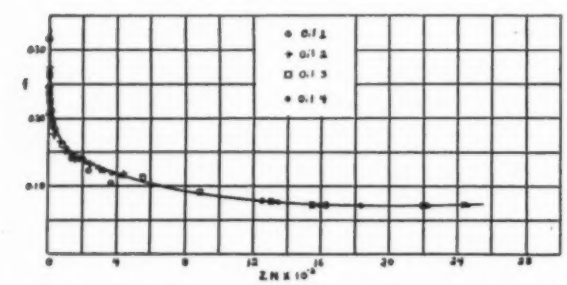

FIG. 7. Curve showing correlation of the data in Fig. 6 by use of the composite variable ZN.

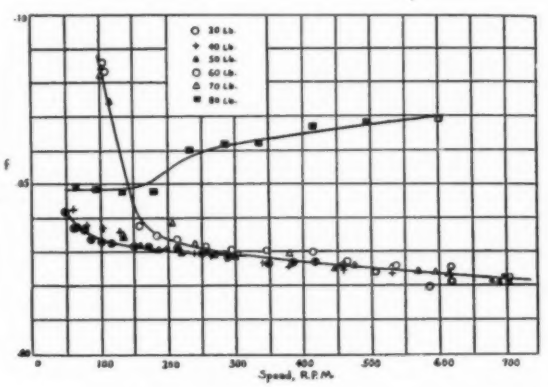

FIG. 8. Coefficient of friction-speed curves for one oil at several loads. Data obtained with a Timken lubricant tester.

changes in viscosity with temperature, but no contrary evidence has been obtained in this work. Some confirmatory data were obtained with a Timken lubricant tester[8] at various temperatures. Briefly, the essential feature of this machine is the rubbing of the surface of a rotating metal cylinder on a stationary flat surface. Results on one oil having a viscosity of 1100 centipoises at 25°C are shown in Fig. 8,

[8] S. A. E. J. **28**, 53 (1931).

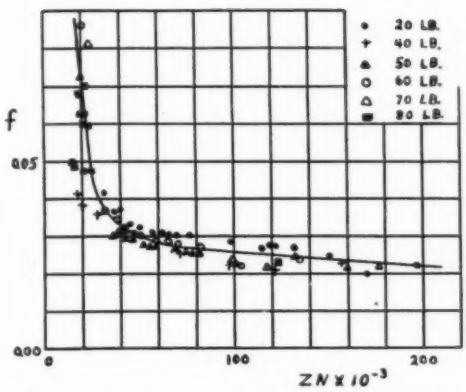

FIG. 9. Curve showing correlation of the data in Fig. 8 by use of the composite variable ZN.

where coefficient of friction-speed curves are given for several loads. Under the conditions of operation with this machine, the coefficient of friction is independent of the load within experimental error. Hence the differences in the coefficient of friction at any constant speed, shown in Fig. 8, are due to changes in oil temperature. From the viscosity-temperature curve of this oil, the viscosity at each temperature was obtained and the coefficient of friction values are plotted in Fig. 9, against ZN. Although the experimental error with this machine is greater than when using the flat plate friction machine, it is seen that the ZN plot appears to account for viscosity effects due to temperature changes.

While the results on the effect of viscosity obtained with three machines in the region of thin film lubrication are admittedly of a preliminary nature, nevertheless they are in agreement in indicating that the effect of viscosity on friction can be accounted for by use of the composite variable ZN. If this conclusion is of general applicability to the type of devices used in practice, it makes possible the investigation of oiliness effects independent of viscosity. Some information on the oiliness of castor and lard oils is given in Fig. 10 where the results were obtained on the flat plate friction machine operated at a constant load. The upper curve represents coefficient of friction values for mineral oils. It is seen that the difference in coefficient of friction between the mineral oils and the fatty oils, involving oiliness, varies with the speed and there is an indication that the three curves would

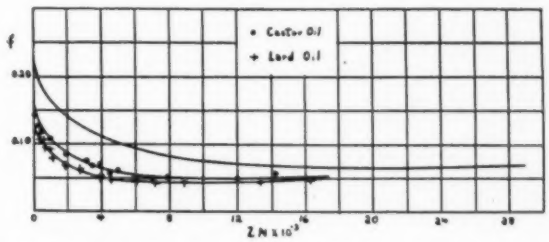

FIG. 10. Coefficient of friction curves for mineral, castor and lard oils.

tend to merge at high values of ZN, in accordance with the information previously obtained in the region of thick film lubrication. Use of the ZN plot makes it possible to compare a fatty oil with a mineral oil of different viscosity and to compare fatty oils of different viscosities.

One point in connection with the generality of a ZN plot should be mentioned. In most practical devices requiring lubrication, the oil film at the entrance to the rubbing surfaces is wedge-shaped and in the machines used, there has been a wedging effect in the oil film. Recently, however, Neely[9] has shown that if a friction machine is purposely designed so as to eliminate any wedging effect, then the coefficient of friction is almost independent of the viscosity over the range investigated by him. Further, it has been observed that if very small amounts of oil are used so that there is little chance for the formation of a wedge in the oil film, the results appear to be almost independent of viscosity. Since a plentiful supply of oil on the rubbing surfaces and the formation of a wedge in the oil film represent the most common case, the present discussion is confined to such conditions.

Assuming therefore that subsequent work will verify the utility of a ZN plot, it will be possible to obtain a practical measure of the effect of oiliness, independent of viscosity over wide ranges of the other variables. These variables appear to be speed, load, temperature and composition of the metal surfaces moving relative to one another. The effect of speed has been considered throughout this discussion and it has been observed that the oiliness effect tends to decrease as the speed is increased.

The effect of load has been mentioned briefly in connection with data obtained on the Timken

[9] Neely, A. P. I. Bull., June 1932, p. 60.

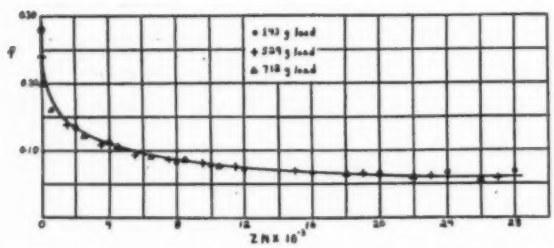

FIG. 11. Coefficient of friction curve for mineral oil illustrating the negligible effect of load in the region of thin film lubrication.

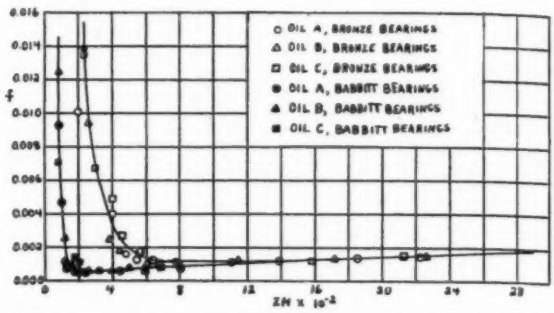
OIL A, BRONZE BEARINGS
OIL B, BRONZE BEARINGS
OIL C, BRONZE BEARINGS
OIL A, BABBITT BEARINGS
OIL B, BABBITT BEARINGS
OIL C, BABBITT BEARINGS

FIG. 12. Curves illustrating the effect of metal composition on the coefficient of friction.

lubricant tester and illustrated in Fig. 9. Results on the flat plate friction machine for three different loads are shown for mineral oils in Fig. 11. The heaviest load corresponds to a pressure of several thousand pounds per square inch. Within experimental error, the coefficient of friction is independent of the load over this range. At high speeds, however, in the region of thick film lubrication, the coefficient of friction varies inversely with the load, so that in the intermediate region between thin and thick film lubrication, the evaluation of the effect of load is complicated. Furthermore, no information has yet been obtained on the effect of load, if any, on oiliness.

No measurements have been made during this investigation on the effect of temperature on oiliness but such measurements should yield information of considerable value for there is evidence that oiliness may vary markedly with temperature. Likewise, differences in oiliness as the result of changes in the composition of the metals rubbing on one another have not been investigated. Some data have been obtained on the differences in coefficient of friction when using metals of various compositions with mineral oils. One example is given in Fig. 12, where the two curves represent data with steel on babbitt and with steel on lead bronze. The major effect is a displacement of the minimum point on the coefficient of friction-speed curve.

Any real discussion of the cause of oiliness must await the acquisition of comprehensive information on the effects of the above-mentioned variables. One thing is very suggestive, however, namely the effect of speed. The fact that the oiliness effect varies with the rate of shear under the high shearing stresses involved, assuming that an extremely thin oil film still exists between the metal surfaces, implies that the phenomenon may be closely linked with viscosity effects. Whether this involves slip or some other characteristic such as interfacial tension is still speculative at present. The whole problem is of great scientific and practical importance and it is hoped that the investigations of the next few years will go far towards an understanding of the fundamental principles of thin film lubrication, about which extremely little information of this nature is available at present.

The Creep of Wires at High Temperature*

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This paper describes apparatus developed to make creep tests on wires at high temperatures in a vacuum, and the results of a series of creep tests on wires of an alloy known as Konal at temperatures of from 800 to 1000°C. It is shown that Ludwik's logarithmic law connecting rate of deformation and stress holds for higher creep rates and stresses, although it certainly does not hold for the lower creep rates and stresses. In the range of temperatures tested, it is found that a linear relation best expresses the variation of the logarithm of the minimum creep rate with temperature. The method used is suggested as being readily adaptable to the making of short time creep tests, for obtaining comparative creep data in a relatively short time on materials used at high temperatures.

THE problem of the creep of metals at elevated temperatures has received a great deal of attention in the last few years due to the increased temperatures used in power plants and industrial processes. Creep[1] may be defined as the slow permanent change in dimensions of materials subjected to the continuous action of stress at elevated temperatures. In most of the experimental work in the literature, the creep has been measured as the extension with time of a bar of material under a constant simple load (tensile stress) distribution at a constant temperature. From the data obtained, materials are compared as to the suitability for high temperature applications, and an attempt is made to obtain data for the design of structures to be operated at high temperatures.

In the tests described in this paper, the material was stressed in tension, and the apparatus was designed with the idea of making fairly rapid tests for the purpose of comparing materials over a range of stress and temperature rather than endeavoring to obtain exact design data. The material tested was an alloy known as Konal[2] which was found by Clark and White[3] to possess great resistance to creep at a temperature of 538°C (1000°F).

TESTS

The apparatus[4] used in these tests was designed for the purpose of making creep tests on wires of about 1.02 mm (0.040 inch) in diameter at temperatures of from 800 to 1000°C at a stress such that sufficient creep would be obtained to be easily measurable in a period of about one week.

In this apparatus (Fig. 1) the four wires to be tested are suspended from cross-pieces on a central column rising from a steel base plate. A weight pan is attached to the lower end of each wire and a dial gauge attached to the base plate measures the lowering of the weight pan and hence the extension of the wire. The loading device with the wires is enclosed under a glass bell jar which can be evacuated by an oil pump and a two stage mercury air pump. The degree of vacuum obtained was of the order of 20×10^{-4} mm of mercury and is measured by a McLeod gauge. The oxidation of the wires at high temperature in this degree of vacuum was found to be negligible by weighing the wires before

* Scientific Paper No. 703.

[1] H. J. Tapsell, *The Creep of Metals*, Oxford University Press, London (1931).

[2] Developed by the Westinghouse Research Laboratories. For further information see E. F. Lowry, *The Rôle of the Core-Metals in Oxide Coated Filaments*, Phys. Rev. **35**, 1367–1378 (1930).

C. R. Austin and G. P. Halliwell, *Some Developments in High Temperature Alloys in the Nickel-Cobalt-Iron System*, Trans. A. I. M. E., Inst. of Metals Div. **99**, 78–96 (1932).

[3] C. L. Clark and A. E. White, *Properties of Non-Ferrous Alloys at Elevated Temperatures.*

[4] The apparatus was designed by Mr. C. E. Crang, formerly of the Westinghouse Research Laboratories, and was later slightly modified by the author.

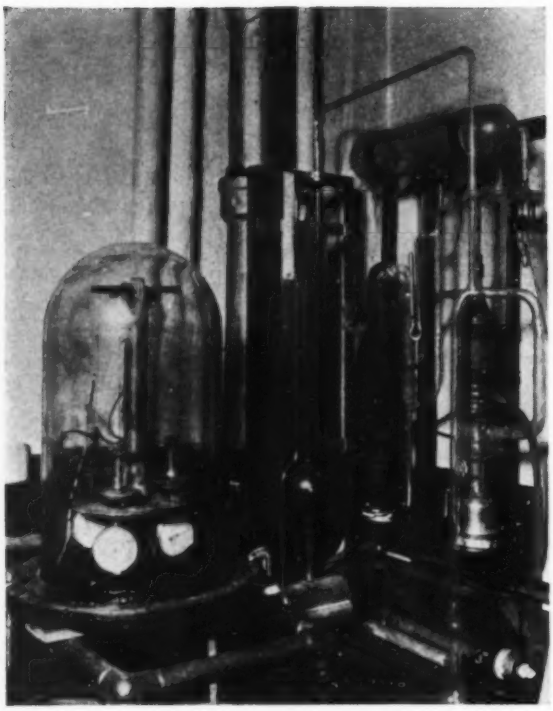

FIG. 1. Photograph of testing equipment.

the test, and at the end of the test after the oxide film had been removed by fine emery paper.

The wires, electrically connected in series, are heated by passing an alternating current through them. The temperature of the wires obtained by this method of heating is quite uniform throughout their length, except for about a tenth of an inch at each end which is cooled by conduction through the grips. In the test an allowance of 0.75 cm (0.3 inch) in the actual gauge length of 23 cm (8.5 inches) was made to take care of this effect. The temperature of the wires is measured by means of an eight mil platinum-platinum rhodium thermocouple spot welded to one of the wires. Optical pyrometer readings taken on all four wires showed them to be at very nearly the same temperature, so that it seemed sufficient to attach a thermocouple to only one wire. Temperature control to within $\pm 5°C$ was obtained by means of a Leeds and Northrup potentiometer temperature controller operating a motor driven resistance.

The procedure of the test was as follows: The wires under appropriate stresses were mounted under the bell jar, which was then evacuated. When a satisfactory vacuum had been obtained, the heating current through the wires was turned on and adjusted until the proper temperature had been reached, as indicated by the thermocouple attached to one wire. Ten minutes after the current was turned on and when the correct temperature had been attained, all four dial gauges were read. These readings served as the zero readings for the test. Afterwards at intervals of about one hour during the day, the dial gauges were again read, the temperature as indicated by the thermocouple being kept constant at the correct value during the readings.

During the preliminary tests it was found that the thermal expansion of the central supporting column of the testing apparatus was of the same order of magnitude in the early part of the test as the creep of the wires and was indicated by the dial gauges as creep. To find the true creep this expansion was measured by means of a micrometer microscope located outside the bell jar and applied as a correction to the dial gauge readings.

MATERIAL

The material used in these tests was cold drawn Konal wire 1.017 mm (0.040 inch) in diameter, fully annealed after drawing.

TEST RESULTS

The results of the tests are obtained, for a given stress and temperature, as a relation between the extension of the wires and time, and these are plotted in the form of creep curves, extensions per unit of length being plotted as ordinates and the time as abscissa. Fig. 2 shows a typical family of such curves for a temperature of 900°C. As has been noticed by other investigators, these curves may be sub-divided into three stages. In the first stage the slope of the curves, or the creep rate is steadily decreasing at a fairly rapid rate. Then a stage occurs in which the creep rate is quite nearly constant, until finally it begins to increase, and then increases up until fracture occurs. If the stress is fairly low the second stage of the creep curve is the predominant one.

Small temperature irregularities in the early part of the test have a large effect on the total

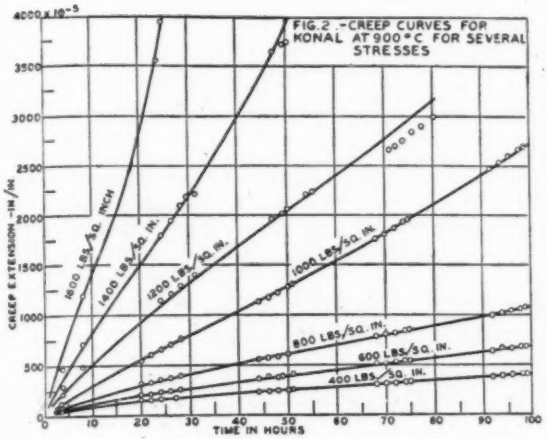
FIG. 2 - CREEP CURVES FOR KONAL AT 900°C FOR SEVERAL STRESSES
CREEP EXTENSION - IN./IN.
TIME IN HOURS
1600 LBS/SQ. INCH
1400 LBS/SQ. IN.
1200 LBS/SQ. IN.
1000 LBS/SQ. IN.
800 LBS/SQ. IN.
600 LBS/SQ. IN.
400 LBS/SQ. IN.

FIG. 2. Creep curves at 900°C.

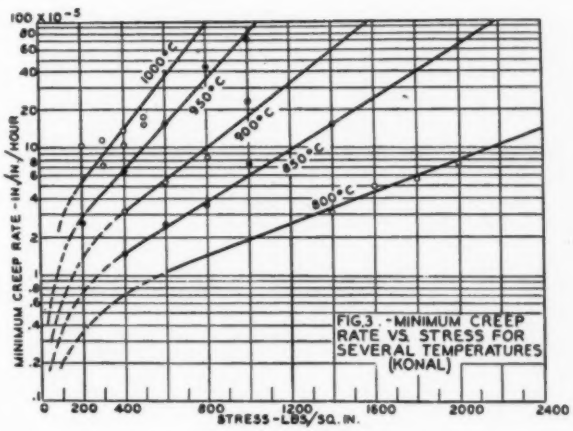

FIG. 3. Min. creep rate *vs.* stress.

extension produced, and so creep curves which are obtained under ostensibly the same conditions of temperature and stress may differ widely. However, these temperature irregularities probably do not affect the slope of the second stage of the creep curve to as large an extent. For this reason, in these tests more attention has been placed on determining the minimum slope of the creep curves, or in other words the minimum rate of creep, than in determining the actual creep curves. According to Tapsell[5] the minimum creep rate represents a definite characteristic of the material under the conditions of the test, and it would seem logical to use it at least for sufficiently small stresses as a basis for comparing materials for use at high temperatures.

The results of the tests are shown in Figs. 3 and 4. Fig. 3 shows the relation between minimum creep rate and stress for several temperatures as plotted on semi-logarithmic paper. It can be seen that for the larger stresses the curves are practically straight, but that they bend down as the stresses become lower and presumably become asymptotic to the zero stress line since there can be no creep rate without an applied stress. The straight portion of the curves would indicate that at a constant temperature Ludwik's logarithmic relation[6] between speed of deformation and stress,

$$\sigma = \sigma_1 + \sigma_2 \log_e (v/v_0),$$

where σ is the applied stress $v = d\epsilon/dt$ is the speed of deformation, and σ_1, σ_2, and v_0 are constants of the material, would hold for sufficiently high stresses and fairly rapid rates of creep, though it certainly would not for the lower ones. This logarithmic relation was originally proposed to express the variation of stress with speed of deformation in cases of plastic flow, and as such has been recently confirmed by Deutler[7] testing copper and steel in tension at normal temperatures. Jamieson[8] has also verified it experimentally by testing lead cylinders in torsion at various straining rates at normal temperature. In these tests on plastic flow of metals it is necessary to consider the stress and speed of deformation at constant amounts of total strain, in order to obtain the logarithmic relation. However, in creep where the creep rate may be considered constant for a range of total strain, this is not necessary. Bailey[9] has used this logarithmic relation between stress and creep rate in some of his analytical work on the creep of tubes and cylinders.

[5] Reference 1, p. 126.

[6] A. Nádai, *The Creep of Metals*, Trans. A. S. M. E., Applied Mech. Div. **1**, 61–67 (1933).

[7] H. Deutler, *Experimentale Untersuchungen über die Abhängigkeit der Zugspannungen von der Verformungsgeschwindikeit*, Phys. Zeits. **33**, 347 (1932).

[8] J. Jamieson, *Influence of Rate of Shear on Shearing Strength of Lead*, Trans. A. S. M. E., Applied Mech. Div. (1933).

[9] R. W. Bailey, *Thick Walled Tubes and Cylinders Under High Pressure and Temperature*, Engineering **129**, 772 (1930).

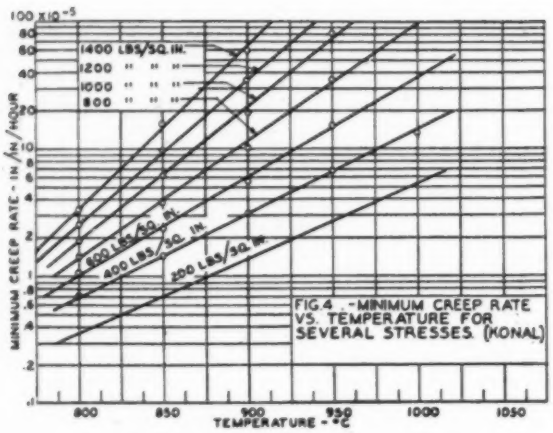

FIG. 4. Min. creep rate *vs.* temperature.

Fig. 4 shows the relation between the minimum creep rate and temperature for a constant stress as plotted on semi-logarithmic paper, and it can be seen that over the range considered, the points for a given stress seem to lie fairly well on a straight line. It will be noticed that the family of lines so defined are not parallel and therefore if produced would meet at some lower temperature of the order of 500°C. This would mean that at some temperature the creep rate for two values of stress would be the same, which is contrary to expectation. It seems probable, therefore, that the linearity shown in Fig. 4 is observed within a comparatively small temperature range such as the one considered, and that for lower temperatures the curves of minimum creep rate will tend to curve downwards and become parallel. This conclusion is borne out by curves showing the variation of creep rate with temperature for a 12 percent chrome iron at somewhat lower temperatures (315 to 760°C) as shown by P. G. McVetty.[10] These curves are concave downwards and appear to become quite nearly parallel at the lower creep rates. However, the curves of Fig. 4 can well be used for interpolation in the range considered in the tests.

ACKNOWLEDGMENT

The author wishes to acknowledge the suggestions and advice received during the progress of the tests and in the preparation of this paper from Dr. A. Nádai and Mr. P. G. McVetty of the Westinghouse Research Laboratories. He is also indebted to Mr. R. E. Peterson, Manager of the Mechanics Division of the Westinghouse Research Laboratories for his interest and support of work of this character.

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[10] P. G. McVetty, *Working Stresses for High Temperature Service*, Trans. A. S. M. E., Applied Mech. Div. (1934).

Concerning the Effect of Strain and Rate of Strain on Tensile Tests at Normal and Elevated Temperatures

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(Received December 13, 1933)

A few idealized cases of plastic flow have been analyzed in an attempt to show the influence of constant coefficients of work-hardening and pure viscosity on tests made at normal and elevated temperatures. Although it is true that the assumed linear expression might approximately fit cases of *very low* stresses and *small strains*, of the order of a percent or less, it cannot be used to explain test results in which annealing and chemical changes occur, as they often do in practice. It is believed, however, that the cases discussed do bring certain fundamental considerations into sharper definition than in the past. They indicate that more emphasis should be placed by experimenters on the changes in yield stress (in the hardness) that occur in materials even without stress under exposure to time and high temperature.

I. INTRODUCTION

RECENTLY investigators have pointed to the desirability of a more detailed fundamental analysis of the mechanical laws governing the slow deformation of metallic materials. The present paper has been written in response to this need with the hope that it will help to clarify conceptions of the influence of strain-hardening acting simultaneously with viscosity in tensile tests of these materials.

In the following treatment it is assumed that the total strain, ϵ, of a distorted body is equal to the sum of an elastic portion, ϵ', proportional to the stress, σ (e.g., $\epsilon'=\sigma/E$, where E is Young's modulus) and a plastic or permanent portion ϵ'', $\epsilon=\epsilon'+\epsilon''$.

Therefore the rates of strain are similarly additive so that, $u=u'+u''$ where

$$u=d\epsilon/dt; \qquad u'=d\epsilon'/dt=d\sigma/Edt; \qquad u''=d\epsilon''/dt.$$

In general, experiments have shown that under sufficiently high temperature metals subjected to constant stress show an increase of deformation with time but that a continual increase in stress is necessary to maintain a constant rate of strain. Thus it is assumed that it is possible to take account of both of these effects by writing,

$$\sigma=F(\epsilon'', u''). \tag{1}$$

This expression must be assumed to apply only where the material does not appreciably soften as a result of the annealing effect of time and temperature. It may be replaced by the tangent plane of the surface given by Eq. (1) for the solution of special cases of physical tests in Section IV. In these applications strains, rates of strain and stresses are considered to be very small. Both theoretical and experimental analysis will be necessary in order to decide whether or not these relations may hold in certain very special cases of interest to the manufacturer. It may be remarked, however, that the designer of high temperature equipment is interested in keeping the total strain well within a percent or a fraction of one percent for an assumed life of a machine part.

II. DEPENDENCE OF STRESS ON STRAIN AND VELOCITY

Although a vast amount of time has been spent in recent years on studies of the plastic flow of metallic materials at elevated temperatures, comparatively little of it has been spent on the fundamental nature of the mechanical laws involved. There is, however, a very serviceable low temperature theory of plasticity, applicable to problems where stresses and deformations may be regarded as independent of time.[1]

Where time and rate of strain must be considered, as in the design of boiler tubes, flanges,

[1] A. Nádai, *Plasticity*, McGraw-Hill Book Co., N. Y. (1931).

and turbine casings, for high temperature service, recourse must be had to the analyses of R. W. Bailey,[2] K. Bauman,[3] G. H. MacCullough,[4] C. R. Soderberg[5] and others.[6] These writers have usually tried to interpret the data obtained from the most common form of creep test, namely a test in which plastic or total strain and time values are recorded while the tensile load (or sometimes torsional stress) and temperature acting on a specimen are kept constant. Such tests commonly give curves of the type shown in Fig. 1. References to tests of this nature are quite numerous but particular mention must be made in this connection to the results of P. G. McVetty,[7] L. W. Spring,[8] F. L. Everett[9] and H. J. Tapsell.[10] Work along similar lines to that found in the present paper has been published by A. Nádai[11] and a number of writers[12] whose investigations have been mostly published in the *Journal of Rheology*.

Most investigators have considered that the primary creep is negligible and that the stress may be assumed to depend only upon the minimum rates of strain or, at sufficiently low stresses, upon the seemingly constant straining rates immediately following the primary creep. In this region the rate of strain is almost wholly plastic. The recently found relation connecting these rates with stress is

$$\sigma = \sigma_1 + \sigma_2 \ln (u''/u_0).$$

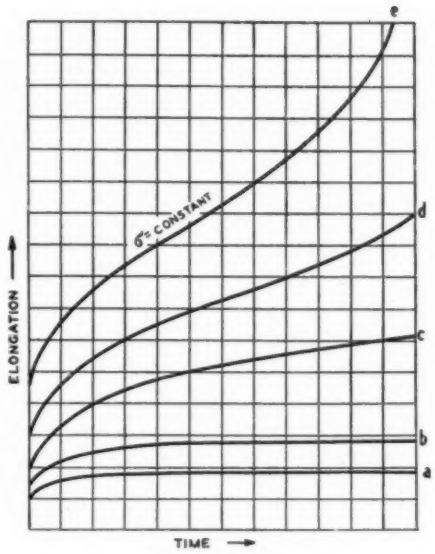

FIG. 1.—CREEP CURVES FOR VARIOUS STRESSES AT SAME TEMPERATURE.

It is assumed that σ_1 and σ_2 are invariable for a constant value of ϵ''. This law does not describe the dependence of stress on velocity as measured by straining a test specimen at various constant rates of deformation since it obviously cannot hold where rates of strain and stress are very small. In the latter case there is some reason to believe, particularly at higher temperatures, that metals act as purely viscous bodies, i.e., that stress is strictly proportional to the rate of plastic deformation.

It is true, however, that the logarithmical dependence of stress on rate of strain seems to have considerable experimental foundation if correctly applied. It was postulated for normal temperatures by P. Ludwik[13] and H. Cassebaum[14] independently, and later confirmed by H. Deutler[15] and others who made tests under normal temperature conditions which showed the tensile stress necessary, at any strain, to maintain a very wide range of constant rates of total strain. Creep investigators, on the other hand, have commonly used the straining velocities of the straight portions of creep curves or minimum creep rates.

[2] R. W. Bailey, Engineering **129**, 265, 327, 772, 785, 818 (1930); **133**, 297 (1932).

[3] K. Bauman, Engineering **130**, 597, 661, 723 (1930).

[4] G. H. MacCullough, Trans. A. S. M. E., A.P.M.—55—9, 12 (1933).

[5] C. R. Soderberg, Trans. A. S. M. E., A.P.M.—55—16 (1933).

[6] See references of Joint Symposium A. S. M. E. and A. S. T. M. (1931).

[7] P. G. McVetty and N. L. Mochel, Trans. A. S. S. T. **2**, 73 (1927). P. G. McVetty, N. L. Mochel and T. D. Lynch, Proc. A. S. T. M. **25**, Part 2 (1925). P. G. McVetty, Trans. A. S. M. E., A.P.M. 55—13 (1933), Abstract before Creep of Metals Session, A. S. M. E. (1933).

[8] L. W. Spring and J. J. Kanter, Proc. A. S. T. M. **30**, 110 (1930).

[9] F. L. Everett, Trans. A. S. M. E., A.P.M. 53—10 (1931).

[10] H. J. Tapsell, *Creep of Metals*, Oxford University Press (1931).

[11] A. Nádai, Trans. A. S. M. E., A.P.M. 55—10 (1933).

[12] See references of J. Rheology **3**, No. 1, 16 (1932).

[13] P. Ludwik, *Elemente der Technel. Mechanik*, Berlin, J. Springer (1908).

[14] H. Cassebaum: Dissertation University of Göttingen (1911), Ann. d. Physik **4**, 34, 106 (1911).

[15] H. Deutler, Phys. Zeits. **33**, 347 (1932).

At the Westinghouse Research Laboratories velocities of points on the different creep curves reached in the same time intervals (i.e., for equal values, $t=$const., of the time t) have also been compared and found, if the rate of strain is not too small, to lead similarly to a logarithmical expression which appears to hold even for the primary portion of the creep curves. The work done to confirm this experimental relation indicates that creep rates taken from the apparently straight portions of ordinary creep curves are practically the same as the creep rates at a certain comparatively large value of strain, perhaps even the same as the minimum creep rates.

III. Stress Surface

The following analyses assume that at any constant temperature the stress σ depends solely on the plastic strain ϵ'' and the rate of plastic strain u'', i.e.,

$$\sigma=F(\epsilon'', u''). \tag{1}$$

Eq. (1) may be assumed to embody the results of tensile tests in which a metallic specimen of bar stock is stretched at a constant rate of plastic strain. It is assumed that the shape of this surface does not change simply because of the passage of time. Softening of strain-hardened metal caused by annealing, and chemical changes must therefore be excluded. In other words, the times required to make any test such as those considered in subsequent sections must be small compared to the time required for softening or chemical changes to occur.

The surface of Eq. (1) may be replaced by its tangent plane in a limited region. Denoting by ϵ'' and u'' *changes* in the coördinates of the surface from the point of tangency,

$$\sigma=\sigma_1+\psi\epsilon''+\varphi u''. \tag{2}$$

Physically interpreted, $\psi=(\partial\sigma/\partial\epsilon'')_P$ may be called the "coefficient of strain-hardening for tension" and $\varphi=(\partial\sigma/\partial u'')_P$ may be called the "coefficient of viscosity" both being defined at the point of tangency, P.

Writing σ for $\sigma-\sigma_1$ the equation

$$\sigma=\psi\epsilon''+\varphi u'' \tag{3}$$

is a representation of the *change* in stress in the tangent plane corresponding to linear changes of amount ϵ'' and u''. Omitting the $\psi\epsilon''$ term we have $\sigma=\varphi u''$ which has the form of Bingham's law mentioned below, or, if σ_1 be zero, of the law governing pure viscous flow. Replacing u'' by the equivalent expression $d\epsilon/dt-(1/E)(d\sigma/dt)$ the law for pure viscous flow may easily be rewritten in the form used by Clerk Maxwell to describe "independently of hypothesis" the phenomena of viscosity in all bodies. He wrote, saying that φ/E might be a function of σ,

$$d\sigma/dt=E(d\epsilon/dt)-\sigma/(\varphi/E).$$

H. Jeffreys[16] has called this mode of deformation *elasticoviscosity*. Besides this case he has introduced for the elastic state the law common to the theory of vibration

$$\sigma=E\epsilon+\varphi_1 d\epsilon/dt,$$

which he calls the *firmoviscous* law and attributes to Sir J. Larmor. Jeffreys also combines these two expressions thus,

$$E\epsilon+\varphi_1 d\epsilon/dt=\sigma+(E/\varphi)\int\sigma dt.$$

For certain other cases of plastic flow, E. C. Bingham[17] introduced another assumption. He assumed a special case of plastic deformation could be represented by postulating a finite plastic limit such that below a certain value of stress σ_{cr} there could be no plastic deformation while above it the pure viscous flow law $\sigma=\varphi d\epsilon''/dt$ would hold.

In the following four examples are given as illustrations of the consequences of the linear expression $\sigma=\psi\epsilon''+\varphi u''$ (Eq. 3) in which it is assumed that ψ and φ are constants.

(a) The *constant stress* test ($\sigma=$constant).

(b) The *relaxation* test ($\epsilon=\epsilon'+\epsilon''=$constant).

(c) The *constant rate of total strain* test ($d\epsilon/dt=$constant).

(d) The *constant rate of increase of stress* test ($d\sigma/dt=$constant).

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[16] H. Jeffreys, *The Earth . . .*, University Press, Cambridge, 2nd Ed., p. 263 (1929).

[17] E. C. Bingham, Bur. Stand. Sci. Pap. **13**, 309 (1916). J. Rheology **1**, 507 (1930).

IV. Special Cases

(a) Constant stress test (σ = constant)

The expression, ordinary creep test, commonly refers to a test in which the load on a specimen is kept constant while values of strain and time are observed. However, for the small deformations (ϵ'' of the order of one percent and less) with which engineers usually have to deal there is no appreciable difference between a test under constant load and one under constant stress. The latter has been chosen as an example in this section, rather than the constant load test, because it leads to a more compact answer.

Recalling that $u''=d\epsilon''/dt$, Eq. (3) may be directly integrated with σ held constant. If ϵ'' is taken to be zero when t is zero, the integration gives,

$$\epsilon''=(\sigma/\psi)(1-e^{-\psi t/\varphi}). \qquad (4)$$

Hence it is apparent that the assumption of constant coefficients of work hardening and viscosity leads to a finite limiting value of creep $\epsilon''=\sigma/\psi$ for $t=\infty$. As a consequence $d\epsilon''/dt$ curves, plotted as a function of time for each value of constant stress, approach zero as $t\to\infty$.

(b) Relaxation ($\epsilon=\epsilon'+\epsilon''$ = constant)

Relaxation is represented in practice by the gradual loosening of originally tight bolts when exposed to elevated temperatures.

To test this phenomenon in the laboratory a bar with modulus E is strained elastically, and subsequently maintained at a deformation $\epsilon=\sigma_0/E$ where σ_0 is the initial stress. The stress in the bar disappears as the elastic strain changes into plastic strain and is recorded as a function of time. The differential equation describing this change is obtained by simply setting $\epsilon'+\epsilon''=\sigma_0/E$ a constant. In the case at hand, it is,

$$\epsilon'+\epsilon''=\left(\frac{\sigma}{E}\right)+\left(\frac{\sigma}{\psi}-\frac{\varphi}{\psi}\frac{d\epsilon''}{dt}\right)=\frac{\sigma_0}{E}. \qquad (5)$$

This is equivalent to passing a plane parallel to the u'', σ plane through the ordinate representing initial strain in the surface of total strain *versus* stress and rate of plastic strain.

Since $\epsilon''=(\text{constant})-\sigma/E$, the differential equation becomes in terms of σ, σ_0, and t,

$$d\sigma/dt+(E+\psi)\sigma/\varphi=\psi\sigma_0/\varphi, \qquad (6)$$

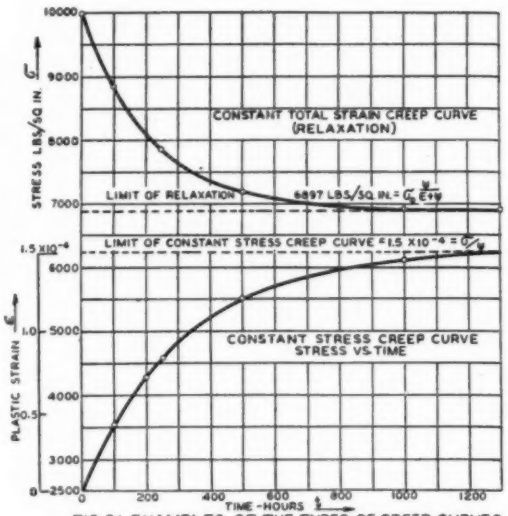
CONSTANT TOTAL STRAIN CREEP CURVE (RELAXATION)
LIMIT OF RELAXATION
6897 LBS/SQ IN.
LIMIT OF CONSTANT STRESS CREEP CURVE
CONSTANT STRESS CREEP CURVE STRESS VS TIME
STRESS LBS/SQ IN
PLASTIC STRAIN
TIME - HOURS

FIG. 2.—EXAMPLES OF THE TYPES OF CREEP CURVES DISCUSSED IN SECTIONS (a) AND (b)

FOR RELAXATION $\sigma=\frac{\sigma_0\psi}{E+\psi}\left(1+\frac{E}{\psi}e^{-\frac{(E+\psi)}{\varphi}t}\right)=6897\left(1+0.45e^{-\frac{2.75}{1000}t}\right)$

FOR CONSTANT STRESS $\epsilon''=\frac{\sigma}{\psi}\left(1-e^{-\frac{\psi}{\varphi}t}\right)=\frac{1.5}{10^4}\left(1-e^{-\frac{2.63}{1000}t}\right)$

—WHERE—

$\sigma=10000$ LBS/SQ IN. (CONSTANT STRESS) $\quad E=30\times10^6$ LBS/SQ IN.

$\sigma_0=10000$ LBS/SQ IN. $\quad \psi=\frac{2}{3}\times10^6$ LBS/SQ IN

$\varphi=2.05\times10^{10}$ LB. HOURS PER SQ. IN.

whence, by integration using the boundary condition $\sigma=\sigma_0$ when $t=0$,

$$\sigma=[\sigma_0/(E+\psi)][\psi+Ee^{-(E+\psi)t/\varphi}]. \qquad (7)$$

It is apparent from this expression that σ approaches a finite limiting value, $\sigma_0\psi/(E+\psi)$, with the passage of time. In the case of pure viscous flow ($\psi=0$) this limiting value is zero. Thus, if a bar, consisting of material following the law given by Eq. (3), is constrained so that the length remains constant, the stress will decrease because of relaxation by the finite amount, $\sigma_0E/(E+\psi)$, just found.

(c) Constant rate of total strain ($d\epsilon/dt$ = constant or $\epsilon=ut$)

By using certain types of testing machines (the so-called screw-type machines driven by an electric motor) in material testing laboratories, tensile tests of bar stock may be conducted so that the heads, and therefore the ends of the test-piece, move apart at nearly constant velocity. It is therefore of particular interest to know the result of applying Eq. (2) to this case. Here

$$\epsilon=\epsilon'+\epsilon''=\sigma/E+\epsilon''=ut \qquad (8)$$

and this substituted in Eq. (2) gives

$$\sigma=\psi(ut-\sigma/E)+\varphi(u-d\sigma/Edt). \quad (9)$$

By integrating, and applying the boundary values $\sigma=0=t$ it is found that,

$$\sigma=\frac{\psi E}{\psi+E}ut+\frac{\varphi E^2u}{(\psi+E)^2}(1-e^{-(\psi+E)t/\varphi}) \quad (10)$$

as a function of u and t. The stress-strain relation is the one usually recorded and is, in this case,

$$\sigma=\frac{\psi E}{\psi+E}\epsilon+\frac{\varphi E^2u}{(\psi+E)^2}(1-e^{-(\psi+E)\epsilon/\varphi u}). \quad (11)$$

As ϵ increases indefinitely the stress depends linearly on ϵ and u. Thus, for large values of ϵ, the stress-total strain-total velocity surface $\sigma=H(\epsilon, u)$ is also a plane.

(d) Constant rate of increase of stress ($d\sigma/dt$ = constant or $\sigma=wt$)

For a comparison with the results discussed in the last section, (c), the case of a tensile specimen subjected to a constant rate of increase of stress will now be considered. Such a test could, for instance, be made by allowing a constant stream of lead shot or water to fall into a container fastened to the test piece by a suitable system of levers.

Substituting wt for σ, Eq. (3) becomes

$$\sigma=wt=\psi\epsilon''+\varphi d\epsilon''/dt. \quad (12)$$

Integrating and fitting to the origin $\epsilon''=0=t$, the expression for ϵ'' as a function of time becomes,

$$\epsilon''=wt/\varphi-(w\varphi/\psi^2)(1-e^{-\psi t/\varphi}) \quad (13)$$

and, as a function of σ,

$$\epsilon''=\sigma/\psi-(w\varphi/\psi^2)(1-e^{-\psi\sigma/\varphi w}). \quad (14)$$

For large values of σ the stress is related linearly to both w and ϵ''.

Because of loosely defined specifications in the past and the great numbers of different kinds of physical tests of materials then (as now) in use one often finds masses of collected data with little or no indication of the form of test, speeds of testing, etc., used to obtain them. There has been a particular looseness in terminology in the high temperature field, as is, for example, the case with the logarithmical expression discussed in Section II. Investigators have not always clearly defined coefficients introduced, points on curves from which these coefficients are to be taken and the kind of tests used to obtain them.

If, for instance, the work-hardening coefficient were defined from the test of this section, (d):

$$\partial\sigma/\partial\epsilon''=\psi/(1-e^{-\psi\sigma/\varphi w}),$$

$$\partial\sigma/\partial\epsilon''|_{\sigma=0}=\infty, \quad \partial\sigma/\partial\epsilon''|_{\sigma=\infty}=\psi.$$

This difference was, of course, to be expected. It would be well to indicate both the point of definition and form of test used to determine coefficients as subscripts to defining partial derivatives as is customary in thermodynamics.

Acknowledgments

The writer is particularly indebted to Dr. A. Nádai of the Westinghouse Research Laboratories for having suggested and assisted this investigation. He wishes, too, to express his appreciation for the many helpful conversations he has had with Mr. P. G. McVetty of the same company.

Acknowledgments are also extended to Mr. L. W. Chubb, Director of the Research Laboratories, and to Mr. R. E. Peterson, Manager of the Mechanics Division, of the Westinghouse Electric and Manufacturing Company for permission to publish the paper.

Concerning the Distribution of Stress* in a Laterally Compressed Strip

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(Received December 13, 1933)

Photoelastic tests are described in which the contact pressure distribution produced by laterally compressing a strip of Bakelite between rollers of the same material is approximately determined. Further, the shapes of the shear stress trajectories as predicted by the elastic theory for a cylinder compressed between two concentrated loads are compared with the shapes of the flow layers obtained by compression tests on mild steel cylinders. A fairly good agreement was obtained.

I. Introduction

WITHIN the last ten years various attempts have been made to determine the stresses produced in a narrow strip by the action of rollers. This subject is of particular importance to the rolling mill engineer and already considerable valuable information is available. The pressure distributions under the rolls which occur during the rolling of a steel sheet have been determined both experimentally and theoretically by von Kármán,[1] Huber,[2] Körber,[3] Siebel[3] and more recently by Lueg.[4] Further observations concerning the action of concentrated pressure in such cases have been reported by A. Nádai,[5] and G. Mesmer.[6]

In most of the above investigations, tests were made with steel sheets under conditions which more or less closely duplicated those actually existing in the rolling mill. These test results therefore refer to cases in which the material has been plastically deformed, with exception perhaps of the work of G. Mesmer. In the discussion to follow no attempt has been made to analyze the dynamic or static equilibrium of stresses occurring in the actual rolling mill while the material is being plastically deformed, or to try to duplicate true conditions in a model test existing under such a process. The purpose of this paper is to describe photoelastic experiments made to determine approximately the elastic pressure distribution in a Bakelite model test bar of about the same shape as a rolled sheet and to compare it with the pressure distribution found in rolled metal sheets by Lueg. It is realized that true rolling conditions are in many respects quite different. It is hoped, however, that by these experiments some further light may perhaps be thrown on the subject of pressure distributions in a rolled sheet. Further, tests on mild steel cylinders will be described in which the flow-layers (Lueder's lines) obtained are compared with the shear stress trajectories predicted theoretically in an elastic cylinder compressed by two concentrated forces.

Notation

a—radius of disk, or half-height of strip.

t—thickness of disk or strip.

P, P_i—concentrated loads.

$p(\xi)$—contact pressure distribution.

ξ—distance from y axis to any value of pressure $p(\xi)$.

c_{ij}—influence constant for load P_i at point x_j on ox of Fig. 7(a).

R—resultant applied load.

μ, ν—rectangular coordinates of a point in (μ, ν) plane.

b—arbitrary constant.

* Scientific Paper No. 710.

[1] von Kármán, Zeits. f. Angew. Math. und Mech. **5**, 139 (1925).

[2] K. Huber, Zeits. f. Angew. Math. und Mech. **9**, 454 (1929).

[3] Körber and Siebel, Mitt. aus dem Kaiser Wilhelm Inst. f. Eisenforshung **10**, 15 (1928). See also other papers by both authors.

[4] E. Siebel and W. Lueg, Mitt. aus dem Kaiser Wilhelm Inst. f. Eisenforschung **15**, 1 (1933).

[5] A. Nádai, *Plasticity*, p. 245, McGraw-Hill, New York (1931).

[6] G. Mesmer, *Inaug.-Diss.*, Göttingen, Feb., 1929.

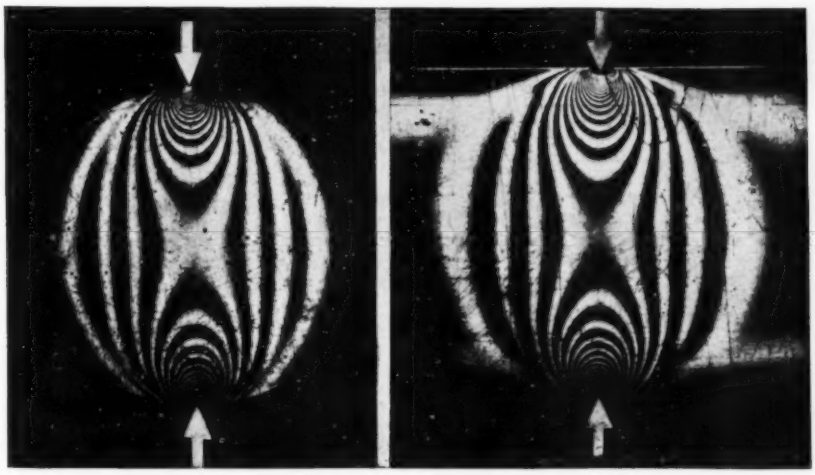

FIG. 1. FIG. 2.

FIG. 1. Fringe photograph of disk compressed laterally under a load of 65.3 kg (144 lbs.).
FIG. 2. Fringe photograph of rectangular strip compressed laterally under a load of 82.1 kg (181 lbs.).

II. COMPARISON BETWEEN THE DISK AND FLAT STRIP

The exact solutions for the determination of the stresses in the elastic state for the cases of the disk and the flat strip compressed between two concentrated loads are already known.[7] For the purpose of comparing the distributions of isochromatics or lines of constant maximum shearing stress for these cases, two photoelastic tests were made. A disk 3.81 cm (1½ inches) in diameter and a strip 3.81 cm (1½ inches) wide by 15.24 cm (6 inches) long were cut from a sheet of Bakelite, compressed between rollers in a special loading frame and photographed in monochromatic light provided by a mercury vapor lamp. Fig. 1 shows the isochromatics for the disk under a load of 65.3 kg (144 lbs.) and Fig. 2 represents the same for the strip under a load of 82.1 kg (181 lbs.). From these figures an interesting observation may be drawn, namely that the shape of the isochromatic lines in both cases is very similar and that the distributions of the differences of the principal stresses are practically the same within the region shown in the figures. A further comparison of the two cases of the disk and the strip can be made by referring to Fig. 2(a). In this figure, the distributions of stresses along the center section ox (of Fig. 7(b)) midway between the loads as computed for the strip by Seewald and as calculated by the disk theory are compared. It is seen that the vertical stress σ_y is very nearly the same for both cases. This fact is later utilized in evaluating stresses along the centerline of more complicated test bars.

III. APPROXIMATE PRESSURE DISTRIBUTION FOR A FLAT BAR COMPRESSED BETWEEN TWO ROLLS

A Bakelite test bar of the shape indicated in Fig. 3 was machined from the same Bakelite sheet used in the former experiments, compressed laterally between Bakelite rollers to a load of 124.7 kg (275 lbs.) and photographed in the photoelastic apparatus using monochromatic light. The result is shown in Fig. 4. Tensile loads were next added acting on the bar shown in Fig. 3 towards the right, at the same time maintaining the original amount of lateral compression. Fig. 5 illustrates the isochromatics received in this test bar with a load of 20.4 kg (45 lbs.) in the axial direction superimposed on a load of 124.7 kg (275 lbs.) in the lateral direction. In Fig. 6 the axial tensile load was increased to 22.7 kg (50

[7] For work on the disk since the work of H. Hertz, and for work on the strip recently due to F. Seewald, see Abhand. aus dem Aerodynamischen Inst. an der Techn. Hochschule, Aachen **7**, 11 (1927) and G. Mesmer, ref. 6.

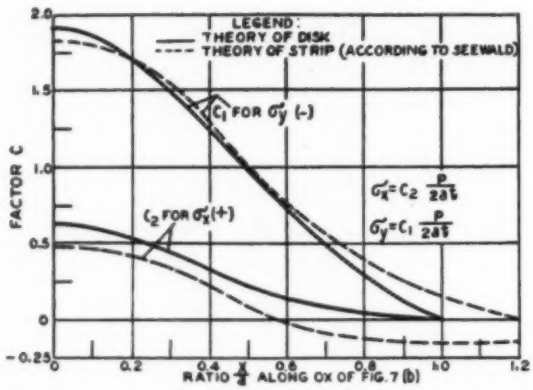
LEGEND:
THEORY OF DISK
THEORY OF STRIP (ACCORDING TO SEEWALD)
C1 FOR σy (-)
C2 FOR σx (+)
σx = C2 P/2at
σy = C1 P/2at
FACTOR C
RATIO x/a ALONG OX OF FIG. 7 (b)

FIG. 2a. Comparison of stresses along center line midway between loads for disk and strip.

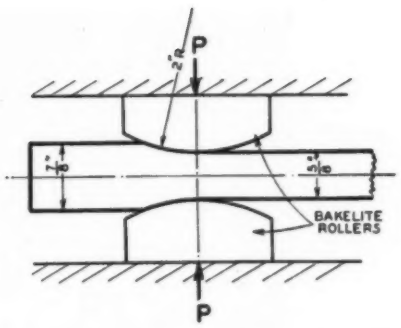

FIG. 3. Schematic sketch of roller compression test on Bakelite bar. Thickness of specimen 11/32".

lbs.), the lateral load remaining the same. These latter two examples were only included to indicate what happens when axial tensile loads are also acting in addition to lateral compression, such as occurs in the Steckel mill.[8]

Since the radii of the Bakelite rolls used are fairly large and the percent reduction not too great, the problem of determining the contact pressure distribution between the rolls and the strip in the case of Fig. 4 can, as a first approximation, be reduced to that shown in Fig. 7(a). In making this simplification, any shear stresses present along the boundary of the contact area are neglected as well as the fact that for a certain portion of the contact area the loads are not strictly vertical but are normal to the surface. In the latter figure the strip with a changing cross section compressed between rolls is replaced by one with a uniform section having a height $2a$ which is the average of the smallest and largest heights shown in Fig. 4, and which is acted upon by a certain unknown contact pressure distribution $p(\xi)$. From the results of Seewald,[7] it is known that the vertical stress σ_y along the centerline ox of a strip compressed between two concentrated loads P, as indicated in Fig. 7(b), can be expressed by a function $G(x/a)$ as

$$\sigma_y = (P/2at)G(x/a). \tag{1}$$

It can easily be shown that similarly for the case illustrated in Fig. 7(a), the vertical stress σ_y along the centerline ox can be expressed by

$$\sigma_y = \varphi(x) = \frac{1}{2a}\int_{\xi_1}^{\xi_2} G\left(\frac{x-\xi}{a}\right)\cdot p(\xi)\cdot d\xi, \tag{2}$$

which is a linear integral equation of the first kind. In Eq. (2), with the procedure to follow,

[8] For a discussion of this process, see E. Siebel, Maschinenbau **11**, 500 (1932).

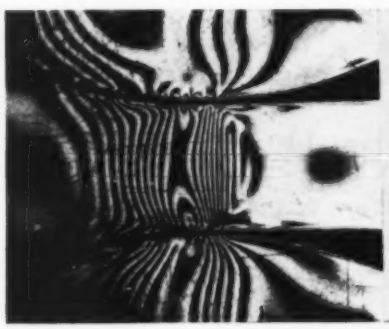

FIG. 4.

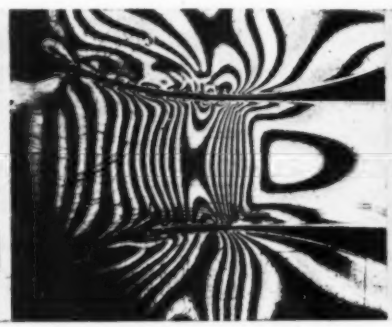

FIG. 5.

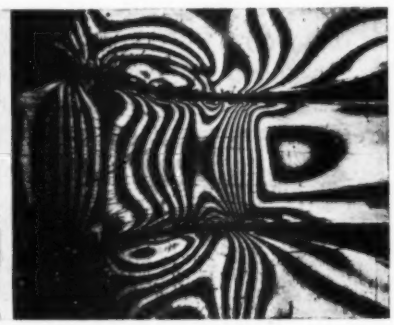

FIG. 6.

FIG. 4. Isochromatics produced by compressing Bakelite bar shown in Fig. 3 to 275 lb. load.
FIG. 5. Isochromatics produced by adding 45 lbs. axial tension to bar loaded as in Fig. 4.
FIG. 6. Isochromatics produced by adding 50 lbs. axial tension to bar loaded as in Fig. 4.

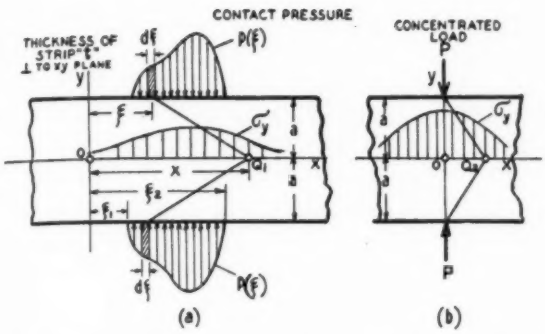

FIG. 7. (a) Strip with uniform cross section under the action of contact pressure $p(\xi)$ replacing the strip with varying cross section of Fig. 4; (b) uniform strip under the action of concentrated loads P.

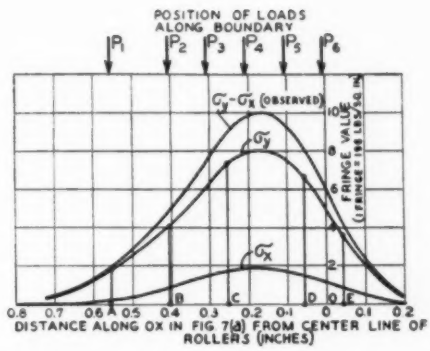

FIG. 8. Distribution of stresses along center line ox of test bar shown in Fig. 4.

the kernel $G[(x-\xi)/a]$ and $\varphi(x)$ will be shown to be known approximately, while $p(\xi)$ is unknown and is the function which is to be determined.

The solution of Eq. (2) for practical purposes may be found by adopting a procedure somewhat similar to that used in the theory of integral equations in which the integral equation itself is replaced by a system of linear equations containing a number of the unknown values of the function to be determined. Following this method, the area under the continuous pressure distribution is next to be divided into n parts, a concentrated load P_i acting at the center of each division replacing the action of the distributed pressure over that division. In order to determine these n loads it is necessary to have n linear equations, one of which is already given since the sum of the loads must equal the resultant applied load. It is thus necessary to know the values of σ_y at $n-1$ points along ox. Eq. (2) may then be replaced by

$$(\sigma_y)_{x=x_j} = (1/2at) \sum_{i=1}^{n} c_{ij} P_i \qquad (j=1, 2, \cdots, n-2, n-1), \qquad (3)$$

$$R = \sum_{i=1}^{n} P_i,$$

which are n linear equations in the n unknown loads P_i.

Eqs. (3) contain a number of influence constants c_{ij} as well as values of σ_y at the points x_j along ox. These quantities are in reality individual values of the functions $G[(x-\xi)/a]$ and $\varphi(x)$, respectively. By combining known theory with some experimental information, it can be shown that approximate values may be obtained for these quantities. Because of limited space, the method used to determine these quantities will only be indicated very briefly.[9] With these

[9] As has already been shown in Figs. 1 and 2, the distributions of the constant principal shearing stresses for the disk and strip compressed laterally are practically the same. Using Seewald's results, Körber and Siebel have shown that the values of σ_y for the disk and for the strip for point o of Fig. 7(b) are very nearly equal (within 4.5 percent). From Fig. 2(a), it may be seen that also for other points along ox, the values of σ_y computed, using the theory of the disk, agree very well with those determined from the exact theory of the strip. These facts enable us to make use of the disk theory in calculating stresses along ox for the bar shown in Fig. 4. With this information available an approximate distribution of σ_y or $\varphi(x)$ along ox of Fig. 7(a) was obtained as follows:—Five unknown loads were taken at arbitrary points along the contact area, and after obtaining a rough value of σ_y by subtracting σ_x as computed from the disk theory using the resultant load of 275 lbs. from $\sigma_y-\sigma_x$ observed which was obtained from Fig. 4, these unknown loads were determined by making use of the function $G(x/a)$ shown in Eq. (1). The latter function was obtained as a first approximation by subtracting the influence function as computed for the disk theory for σ_x from the influence function for $\sigma_y-\sigma_x$ as observed on the strip of Fig. 2. With these approximate loads thus determined, the values of σ_x along ox produced by each were next computed by the disk theory. These values were then super-imposed and the resultant σ_x computed in this manner was then arbitrarily reduced by 25 percent since it is known that the disk theory gives 25 percent higher stresses for certain points along ox than the exact theory of the strip under the same load. This corrected distribution of σ_x is shown in Fig. 8. By subtracting this from $\sigma_y-\sigma_x$ observed a fairly accurate distribution of σ_y along ox was obtained

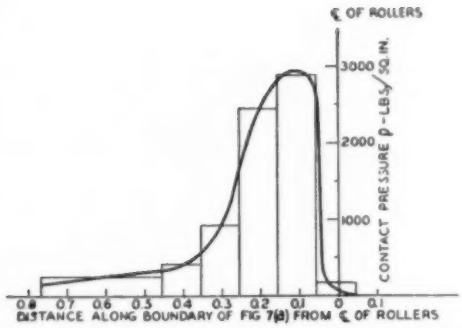
℄ OF ROLLERS
3000
2000
1000
CONTACT PRESSURE p-LBS/SQ IN.
0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0 0.1
DISTANCE ALONG BOUNDARY OF FIG 7(a) FROM ℄ OF ROLLERS

FIG. 9. Approximate elastic contact pressure distribution along boundary of bar shown in Fig. 4 produced by Bakelite rollers.

values of c_{ij}, and σ_y at x_j thus determined approximately,[9] the next step was to select a number of positions on the boundary for the unknown loads P_i. To simplify the work, only six unknown loads were taken along each boundary of Fig. 7(a) located at the positions indicated in Fig. 8. Hence $n=6$ in Eqs. (3). Five points (A, B, C, D, E) along the centerline ox were next chosen, and the influence of each load at these points (or c_{ij}) obtained as explained before from the disk theory. As shown in Fig. 8, the resultant stress σ_y due to all six loads at each point is now known. Hence, Eqs. (3) were then solved for the unknown loads P_i. The loads so determined were then divided by their respective areas and the average stresses obtained for each section plotted by means of the horizontal lines shown in Fig. 9. A smooth curve was then drawn through these lines giving the same resultant load as provided by the rectangular areas. This curve then represents approximately the pressure distribution between the rolls and strip of Fig. 4.

Recently, tests have been made to determine the contact pressure distribution in the rolling mill under true service conditions by Siebel and Lueg.[4] In their investigation, pressures were measured by means of piezo-crystals, and the tests were conducted on thin stips having a thickness about 4 percent of the radial dimension of the rolls. The ratio of dimensions which they

which checks with the resultant load within 2.9 percent. Values for c_{ij} appearing in Eqs. (3) were next determined from the disk theory, which can be done fairly accurately for σ_y according to the above observations.

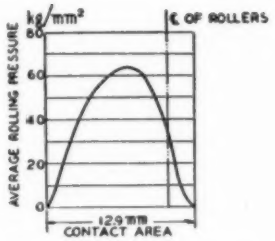

FIG. 10. Contact pressure distribution with the rolling of copper strip. (According to Siebel and Lueg); radius of roller, 90 mm; dimensions of strip, 4×30 mm; reduction, 25 percent.

used was thus considerably different than that used in the photoelastic tests described herein. However, a curve showing the variation in contact pressure which they obtained in cold-rolling copper strip is shown in Fig. 10 which may be compared with Fig. 9.

IV. TESTS WITH MILD STEEL

Two different tests which are somewhat closely connected with the rolling problem were next made on mild steel bars for the purpose of illustrating the type of flow layers (Lueder's lines) obtained in such cases.

In order to show the shape of the boundary of the plastic region in a bar compressed as before between two rollers, a flat bar of mild steel was next machined as shown in Fig. 11. It was then compressed as indicated in this figure, between four semi-cylinders, to a load of about 36,900 kg (80,000 lbs.) per vertical pair of cylinders, and the resultant flow layers on

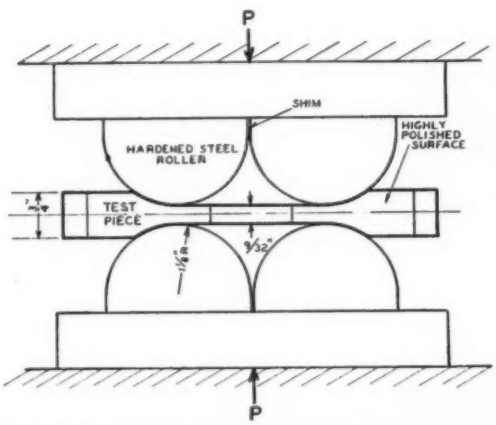

FIG. 11. Roller compression test of mild steel bar. Test piece $5\frac{3}{4}$" long×$2\frac{3}{8}$" wide.

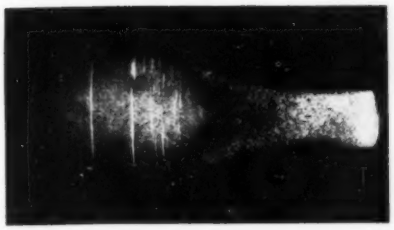

FIG. 12. Flow layers (Lueder's lines) received by compressing the flat bar shown in Fig. 11.

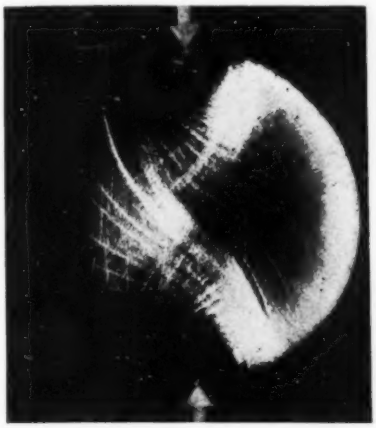

FIG. 13. Flow layers obtained by compressing a mild steel cylinder laterally between concentrated loads.

one of the side faces of the bar photographed by the Schlieren method. The type of flow layers received are shown in Fig. 12.

Next, a mild steel cylinder having polished plane sections was compressed laterally along two generatrices of the cylinder until flow layers were clearly visible over the plane polished end surfaces. A Schlieren photograph of this cylinder is shown in Fig. 13. It has been known since the work of Michell that the principal stress trajectories for this case are comprised of the family of bi-centric circles passing through the points of application of the load and their orthogonals as indicated in Fig. 14. To construct the shear stress trajectories or the lines along which some ductile materials are found to yield, it is only necessary to draw the 45° trajectories

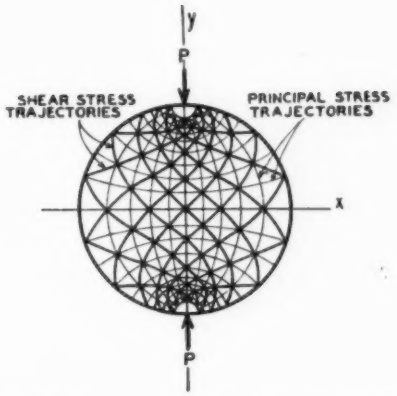

FIG. 14. Principal stress and shear stress trajectories for a disk compressed between two loads. (Elastic theory.)

of this family of circles. These are also shown in Fig. 14. One may easily obtain the equation of these shear stress trajectories by means of conformal mapping.[10] If one compares the shapes of the flow layers in Fig. 13 with the shear stress trajectories of Fig. 14, it appears that first yielding in a laterally compressed cylinder occurs along a family of surfaces, the traces of which follow somewhat the shear stress trajectories predicted from the elastic theory.

ACKNOWLEDGMENT

The author is particularly indebted to Dr. Nádai for his aid and suggestions during the progress of this investigation, as well as to Mr. R. E. Peterson for his interest and support of the work.

[10] This can be done by mapping conformally from a (μ, ν) plane on to the (x, y) plane by means of the relations

$$x = (a \sin \nu)/(\cosh \mu - \cos \nu), \quad (4)$$

$$y = (a \sinh \mu)/(\cosh \mu - \cos \nu), \quad (5)$$

in which a is the radius of the cylinder, and μ, ν are related to each other through the linear equation

$$\nu = \pm\mu \pm b,$$

where b is an arbitrary constant. The expressions (4) and (5) are in reality the parametric equations of the shear stress trajectories.

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